Thus, in (11),

$$p = -x/z, \qquad q = -y/z, \qquad \sqrt{1+p^2+q^2} = a/z,$$

$$\therefore \quad M = -x/a, \qquad N = -y/a,$$

and (11) becomes $k + 2/a = 0$; and (12), $k + 1/b = 0$.

Note 2.—In the foregoing solution I have utilized the notation employed in the chapter on the Calculus of Variations in TODHUNTER's *Integral Calculus*, fifth edition.

374. Proposed by C. N. SCHMALL, New York City.

Show that, on a *Mercator's Chart*, a great circle of a sphere of radius r_1 will be represented by a curve whose equation is of the form

$$c(e^{y/r} - e^{-(y/r)}) = 2 \sin\left(\frac{x}{r} + \theta\right).$$

I. SOLUTION BY ELIJAH SWIFT, University of Vermont.

If the latitude and longitude on the above sphere be denoted by the letters φ and θ respectively, θ varying from 0° to 360°, and φ from $-90°$ to $+90°$; then if axes be taken with origin at the center of the sphere with the xy-plane as the plane of the equator, and if longitude be measured from the x-axis, we have for any point on the sphere

$$x = r \cos\varphi \cos\theta, \; y = r \cos\varphi \sin\theta, \; z = r \sin\varphi.$$

The equation of a great circle is obtained by substituting these values in the equation of any diametral plane, $Ax + By + Cz = 0$, and is

$$(1) \qquad A \cos\varphi \cos\theta + B \cos\varphi \sin\theta + C \sin\varphi = 0.$$

The sphere is mapped on a *Mercator's Chart* by taking a cylinder tangent to the sphere along the equator and projecting a meridian (θ = const.) on a generating line of the cylinder.

Any point on the sphere on this meridian has for its image on the chart a point on the corresponding generating line at a distance $r \log \tan(\pi/4 + \varphi/2)$ from the equator.

When we develop the cylinder on a plane, we can choose axes in that plane so that the coördinates of this point are

$$x = r\theta, \qquad y = r \log \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right).$$

Solving these equations for θ and φ, $\theta = x/r$, $\varphi = 2 \arctan(e^{y/r}) - \pi/2$. Substituting these values in (1), we obtain

$$A \cos\frac{x}{r} + B \sin\frac{x}{r} + C \tan\left\{2 \arctan e^{y/r} - \frac{\pi}{2}\right\} = 0,$$

which reduces at once to the form given, if we let $c = -C/\sqrt{A^2+B^2}$, and $\sin\theta = A/\sqrt{A^2+B^2}$.

II. SOLUTION BY A. M. HARDING, University of Arkansas.

The equation of the sphere is

$$\frac{x}{r} = \cos u \cos v, \qquad \frac{y}{r} = \cos u \sin v, \qquad \frac{z}{r} = \sin u.$$

Any plane through the center will be given by $ax + by + cz = 0$, or

$$a \cos u \cos v + b \cos u \sin v + c \sin u = 0.$$

Dividing by $\sqrt{a^2 + b^2}$, we obtain $\sin (v + \theta) + (c/\sqrt{a^2 + b^2}) \tan u = 0$, where θ is defined by the equations; $\sin \theta = a/\sqrt{a^2 + b^2}$, $\cos \theta = b/\sqrt{a^2 + b^2}$.

The transformation is

$$\frac{y}{r} = \operatorname{logtan}\left(\frac{u}{2} + \frac{\pi}{4}\right), \qquad \frac{x}{r} = v.$$

Solving the first of these equations for $\tan u$, we obtain

$$\tan u = \frac{e^{y/r} - e^{-y/r}}{2}.$$

Hence, by substitution, we have

$$-\frac{c}{\sqrt{a^2 + b^2}}(e^{y/r} - e^{-y/r}) = 2 \sin\left(\frac{x}{r} + \theta\right),$$

which is of the required form.

Also solved by PAUL CAPRON and the PROPOSER.

MECHANICS.

278. Proposed by A. M. HARDING, University of Arkansas.

A spherical shell of mass m explodes when moving with negligible velocity at a height of h feet above the ground. The shell is divided into very small particles, each of which moves, after the explosion, away from the center of the shell with a speed v, and ultimately falls to the ground. Find the total mass of the fragments which will be found per unit area at any specified distance from the point vertically underneath the shell.

SOLUTION BY H. S. UHLER, Yale University.

Let θ and φ denote the angles which the two trajectories, passing through the same point on the ground, make at the instant of the explosion, with the negative and positive directions of the axis of h respectively. φ must be acute but θ may be obtuse. The familiar equation $x = vt + \frac{1}{2}at^2$ leads to

$$r = \frac{v \sin \theta}{g}(+\sqrt{v^2 \cos^2 \theta + 2gh} - v \cos \theta), \tag{1}$$

$$r = \frac{v \sin \varphi}{g}(+\sqrt{v^2 \cos^2 \varphi + 2gh} + v \cos \varphi). \tag{2}$$

Rationalization of (1) and (2) gives

$$gr^2 + 2rv^2 \sin \theta \cos \theta - 2hv^2 \sin^2 \theta = 0, \tag{3}$$

$$gr^2 - 2rv^2 \sin \varphi \cos \varphi - 2hv^2 \sin^2 \varphi = 0. \tag{4}$$